

Axiomatic set theory and its applications

last update:2018–10–26

November 5 (Mon)	
$10:00 \sim 10:10$	opening
Morning Session	Chair:Hiroaki Minami
10:10~11:10	Dilip Raghavan (National University of Singapore) minicourse part 1: TBA
11:20~12:00	David Chodounský (Institute of Mathematics of the Czech Academy of Sciences) Free sequences in $\mathcal{P}(\omega)/fin$
12:00~13:30	Lunch and discussion break
afternoon Session	Chair:Brendle Jörg
13:30~14:10	Teruyuki Yorioka (Shizuoka University)
	The difference between two properties of Aronszajn trees
14:20~15:00	David Aspero (University of East Anglia)
	The special $\aleph_2\text{-}\mathrm{Aronszajn}$ tree property and GCH
15:20~16:00	Yasuo Yoshinobu (Nagoya University)
	**-tactically closed forcing and the Mapping Reflection Principle $% \mathcal{A}^{(n)}$
16:10~16:50	Tadatoshi Miyamoto (Nanzan University)
	A Partition Relation Forced by Side Condition Method

November 6 (Tue)	
Morning Session	Chair:Teruyuki Yorioka
$9:30 \sim 10:10$	Daisuke Ikegami (Shibaura Institute of Technology)
	On preserving AD via forcings
10:20~11:20	Dilip Raghavan (National University of Singapore)
	minicourse part 2: TBA
11:30~12:10	Yo Matsubara (Nagoya University)
	On Countable Stationary Towers
12:10~13:30	Lunch and discussion break
afternoon Session	Chair:Daisuke Ikegami
$13:30 \sim 14:00$	Kenta Tsukuura (Tsukuba University)
	Prikry type Forcing and True Cofinal Sequence
14:10~14:50	Vincenzo Dimonte (University of Udine)
	Left distributive algebras beyond I0
$15:10{\sim}15:50$	Monroe Eskew (Kurt Gödel Research Center)
	Global Chang's conjecture and singular cardinals
16:00~16:40	Sean Cox (Virginia Commonwealth University)
	Some problems related to internal approachability
November 7 (Wed)	
Morning Session	Chair:Brendle Jörg
9:30~10:10	Hiroshi Sakai (Kobe University)
	On generalized notion of higher stationarity in $\mathcal{P}_{\kappa}(\lambda)$
10:20~11:20	Dilip Raghavan (National University of Singapore)
	minicourse part 3: TBA
11:30~12:10	Ulises Ariet Ramos García
	(National Autonomous University of Mexico, Morelia)
	Some applications of iterated ultrapowers in countably compact groups
12:10~	Lunch, discussion and excursion

November 8 (Thu)	
Morning Session	Chair: David Chodounský
9:30~10:10	André Ottenbreit Maschio Rodrigues (Kobe University)
	Some reflection principles at large cardinals
10:20~11:00	Viera Šottová (Charles University)
	Sequence selection principle $S_1(\mathcal{P}, \mathcal{R})$: The critical cardinality
11:20~12:00	Francesco Parente (the University of East Anglia)
	On regularity of ultrafilters and Tukey maximality
12:00~14:00	Lunch and discussion break
afternoon Session	Chair:
$14:00 \sim 14:40$	Miguel CARDONA (TU Wien)
	$<\kappa\text{-uf-extendable matrix iterations}$
15:00~15:40	Diego Mejía (Shizuoka University)
	Many different uniformity numbers of Yorioka ideals
16:00~16:40	Tristan Bice
	(Institute of Mathematics of the Polish Academy of Sciences)
	C*-algebras With and Without <<-Increasing Approximate Units
16:40~	closing

Minicourse

TBA Dilip Raghavan (National University of Singapore) Mon 10:10~11:10, Tue 10:20~11:20, Wed 10:20~11:20

November 5 (Mon)

TBA(Minicourse Part 1) Dilip Raghavan (National University of Singapore) 10:10~11:10

Free sequences in $\mathcal{P}(\omega)/fin$ David Chodounský (Institute of Mathematics of the Czech Academy of Sciences) 11:20~12:00

A free sequence in a Boolean algebra is a sequence of its elements $\langle a_{\alpha} : \alpha < \gamma \rangle$ such that for each $\beta \leq \gamma$ the system $\{a_{\alpha} : \alpha < \beta\} \cup \{a_{\alpha} : \beta \leq \alpha < \gamma\}$ is centered. A free sequence F is maximal if no proper end-extension of F is a free sequence. Don Monk investigated cardinalities of maximal free sequences and defined the free sequence number as the minimal such cardinality. We are mainly interested in the case of the Boolean algebra $\mathcal{P}(\omega)/fin$. The presented results are joint work with J. Grebik and V. Fischer.

The difference between two properties of Aronszajn trees Teruyuki Yorioka (Shizuoka University) 13:30~14:10

The special ℵ₂-Aronszajn tree property and GCH David Aspero (University of East Anglia) 14:20~15:00

Assuming the existence of a weakly compact cardinal, we build a forcing extension in which GCH holds and every \aleph_2 -Aronszajn tree is special. This answers a well-known question from the 1970' s. I will present the proof of this theorem, with as many details as possible. This is joint work with Mohammad Golshani.

**-tactically closed forcing and the Mapping Reflection Principle Yasuo Yoshinobu (Nagoya University) 15:20~16:00

A Partition Relation Forced by Side Condition Method Tadatoshi Miyamoto (Nanzan University) 16:10~16:50

We represent a consistency proof of a partition relation studied by S. Todorcevic by side condition method.

November 6 (Tues)

On preserving AD via forcings

Daisuke Ikegami (Shibaura Institute of Technology) 9:30~10:10

It is known that many concrete forcings such as Cohen forcing destroy AD. In this talk, we show that one cannot preserve AD via forcings as long as the forcing increases Θ and V satisfies AD⁺ and V = L(P(R)). We also provide an example of forcings which preserve AD while increasing Θ when V is not of the form L(P(R)). This is joint work with Nam Trang.

TBA(Minicourse Part 2) Dilip Raghavan (National University of Singapore) 10:10~11:10

On Countable Stationary Towers Yo Matsubara (Nagoya University) 11:30~12:10

H. Woodin introduced the notion of stationary tower forcing. Using stationary tower forcing, he proved that the existence of some large cardinals implies various properties of L(R). We relate the precipitousness of the countable stationary tower of height λ to the complete embeddability of this tower to every countable stationary tower of height $> \lambda$. Then we show that precipitousness of some countable stationary towers, without explicit use of strong large cardinals such as Woodin cardinals, implies some regularity properties of sets of reals in L(R).

Prikry type Forcing and True Cofinal Sequence Kenta Tsukuura (University of Tsukuba) 13:30~14:00

We say that a set of regular cardinals A is progressive if min A > |A|. When A is progressive, pcf(A) has some good properties (e.g. $|pcf(A)| < |A|^{+4}$, max pcf(A) exists, pcf(pcf(A)) = pcf(A),...). For getting such property, can we remove the assumption for progressiveness? It is consistent that there is no example that such good properties for pcf(A) fail without assuming progressiveness. In this talk, we construct a pcf(A) which has no such good property with sup A is singular by using Prikry type forcing.

Left distributive algebras beyond I0 Vincenzo Dimonte (University of Udine) 14:10~14:50

The connection between large cardinals and left-distributive algebras is one of the most intriguing success stories of the theory of large cardinals. LD-algebras are algebras with one operator that satisfies the left-distributive law. At first sight, they have nothing to do with large cardinals, as they can be small, countable, even finite. Yet there is a connection: we say that I3 holds if there is an elementary embedding from V_{λ} to itself. It turns out that such embeddings form a free LD-algebra, therefore any result on such an algebra will propagate, thanks to the universal nature of free algebras, to all LD-algebras. But I3 is not the strongest axiom, there is a hierarchy of other axioms above it. What kind of structure will generate the embeddings related to them? Isomorphic or completely different? A thorough study will show that to have a genuinely new structure one has to go far beyond I0, the strongest common large cardinal, into an experimental hierarchy of E_{α}^{0} axioms, and that weak independence properties can depend from the properness or non-properness of the embeddings involved.

Global Chang's conjecture and singular cardinals

Monroe Eskew

$15:10 \sim 15:50$

Foreman asked to what extent a global version of Chang's Conjecture can hold. The speaker showed with Yair Hayut that, relative to a huge cardinal, ZFC is consistent with the statement that for every regular κ and every $\mu < \kappa$, $(\kappa^+, \kappa) \twoheadrightarrow (\mu^+, \mu)$. In light of constraints imposed by GCH, we asked whether a maximal global Chang's Conjecture is consistent, which says that whenever $cf(\kappa) \ge cf(\mu)$, $(\kappa^+, \kappa) \twoheadrightarrow$ (μ^+, μ) . We show here that it is inconsistent. On the other hand, we show it is consistent relative to a supercompact Vopenka cardinal that for all limit ordinals $\alpha < \beta < \omega^{\omega}$, $(\aleph_{\beta+1}, \aleph_{\beta}) \twoheadrightarrow (\aleph_{\alpha+1}, \aleph_{\alpha})$.

Some problems related to internal approachability Sean Cox (Virginia Commonwealth University) 16:00~16:40

In the early 2000's, Foreman and Todorcevic introduced several variants of the notion of internal approachability. Krueger proved that these notions are consistently distinct under various forcing axioms, and also separated some stationary reflection principles involving them. I will discuss some strengthenings of Krueger's results, and how these strengthenings are related to Strong Chang's Conjecture and various results of Fuchino and Usuba. I will also explain how the Foreman-Todorecevic classes are related to several open problems.

November 7 (Wed)

On generalized notion of higher stationarity in $\mathcal{P}_{\kappa}(\lambda)$ Hiroshi Sakai (Kobe University) $9:30 \sim 10:10$

The generalized notion of higher stationarity in a regular uncountable cardinal κ is relevant to topological semantics of modal logic, and, recently, it is studied by set theorists. In this talk, we discuss the generalized notion of higher stationarity in $\mathcal{P}_{\kappa}(\lambda)$. This is a joint work with Brickhill and Fuchino.

TBA(Minicourse Part 3) Dilip Raghavan (National University of Singapore) 10:20~11:20

Some applications of iterated ultrapowers in countably compact groups Ulises Ariet Ramos García (National Autonomous University of Mexico, Morelia) 11:30~12:10

We use the ultrapower construction to produce various examples of countably compact topological groups, answering thus old problems of Comfort and van Douwen. This is joint work with Michael Hrusak, Jan van Mill, and Saharon Shelah.

November 8 (Thu)

Some reflection principles at large cardinals André Ottenbreit Maschio Rodrigues (Kobe University) 9:30~10:10

This is a joint work with Sakaé Fuchino and Hiroshi Sakai.

In this talk, we present some reflection principles and discuss the restrictions they impose on the size of the continuum.

Assuming the existence of two supercomapct cardinals, we construct a model with large continuum where some reflection statement holds down to $< 2^{\aleph_0}$ holds (namely, a variant of the Diagonal Reflection Principle from [1]), while some stronger reflection statement holds down to $\leq 2^{\aleph_0}$ (namely, a version of the Game Reflection Principle from [2]). The later statement cannot hold any lower since it imposes an upper bound on the size of the continuum.

参考文献

[1] S. Cox. The diagonal reflection principle. Proc. Amer. Math. Soc., 140(8):2893–2902, 2012.

[2] B. König. Generic compactness reformulated. Arch. Math. Logic, 43(3):311–326, 2004.

Sequence selection principle $S_1(\mathcal{P}, \mathcal{R})$: The critical cardinality Viera Šottová (Charles University) $10:20 \sim 11:00$

In 1996 M. Scheepers introduced several kinds of selection principles and he described the basic relations among them. It was done mainly for covers of a topological space X but the space of all continuous functions defined on X, denoted cp(X) was investigated as well.

We are especially interested in $S_1(\Gamma, \Gamma)$ which can be modified by ideals on ω . In case of functions we consider ideal convergence which is common nowadays. This way we extend Scheepers' diagram and we try to determine the differences between the original notion and its ideal modification.

It turns out that the cardinal invariant $\lambda(\mathcal{I}, \mathcal{J})$ introduced by J. Supina is very helpful. We proved it is their common critical cardinality. Therefore we study this combinatorial characteristic in its own and we are interested in its relation with other cardinals.

On regularity of ultrafilters and Tukey maximality Francesco Parente (the University of East Anglia) 11:20~12:00

The study of Tukey reducibility between ultrafilters has been the focus of much research in recent years. One of the main outstanding problems, posed by Isbell in 1965, is whether all nonprincipal ultrafilters over the set of natural numbers have the same Tukey type. In my talk, I shall discuss how regularity of ultrafilters can help shed some light on this problem in the context of Boolean algebras.

$< \kappa$ -uf-extendable matrix iterations

Miguel CARDONA (TU Wien)

14:00~14:40 Mejía [?] introduced the notion of <u>Frechet-linkedness</u> to prove that \mathbb{E} and random forcing are σ -Fr-linked, and that no σ -Fr-linked poset adds dominating reals. Moreover, Brendle, the speaker and Mejía [?] showed that θ -F-Knaster posets preserve strong types of unbounded families.

In this talk we construct θ -Fr-Knaster posets via matrix iterations of $< \theta$ -ultrafilter-linked posets (restricted to some level of the matrix) to force that

- (I) the four cardinals associated with $\mathcal{M}(\operatorname{add}(\mathcal{M}), \operatorname{cov}(\mathcal{M}), \operatorname{non}(\mathcal{M}) \operatorname{and} \operatorname{cof}(\mathcal{M}))$ could be pairwise different, which solves [?, Question 6.2],
- (II) the cardinals associated with \mathcal{N} are also pairwise different (though this consistency was already proved in [?]), and
- (III) the four cardinals associated with Yorioka ideal \mathcal{I}_f are pairwise different for any increasing function $f \in \omega^{\omega}$, which solves [?, Question 6.1].

Many different uniformity numbers of Yorioka ideas Diego Mejía (Shizuoka University) 15:00~15:40

We use a countable support product of creature posets to force that there are uncountably many pairwise different uniformity numbers associated with Yorioka ideals. This is a joint work with Lukas Klausner.

C*-algebras With and Without <<-Increasing Approximate Units

Tristan Bice (Institute of Mathematics of the Polish Academy of Sciences)

 $16:00 \sim 16:40 <$ -Increasing approximate units are a basic tool in the theory of separable C*-algebras. We examine their existence or lack thereof in the non-separable setting. Specifically, we show they do always exist in *omega*₁-unital but not *omega*₂-unital C*-algebras. Using Canadian trees, almost disjoint families and Q-sets we are also able to obtain scattered C*-algebra counterexamples and represent them on smaller Hilbert spaces. These scattered C*-algebras are necessarily LF (locally finite dimensional) but not AF (approximately finite dimensional) in the sense of Farah-Katsura. It follows from our results that the existence of separably representable LF but not AF C*-algebras is independent of ZFC (for more details, see arXiv:1707.09287).