$< \theta$-uf-extendable matrix iterations

Miguel Cardona

Technische Universität Wien

joint work with Jörg Brendle and Diego Mejía

RIMS

Axiomatic set and theory and applications

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Consider

$$\mathfrak{b} = \min\{|F| \; F \subseteq \omega^\omega \text{ and } \neg\exists g \in \omega^\omega \forall f \in F(f \leq^* g)\}.$$
$$\mathfrak{d} = \min\{|E| \; E \subseteq \omega^\omega \text{ and } \forall g \in \omega^\omega \exists f \in E(g \leq^* f)\}.$$

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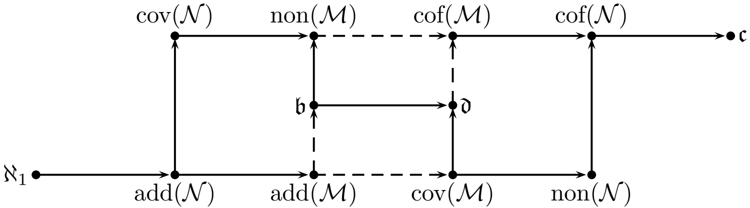

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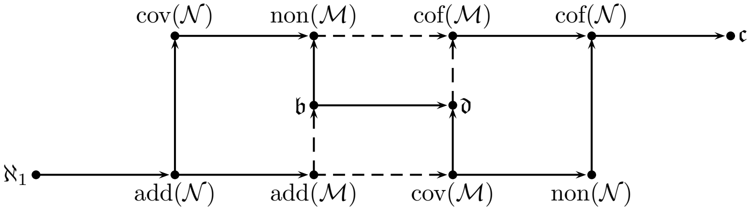

$$\mathrm{add}(\mathcal{M}) = \min\{\mathfrak{b}, \mathrm{cov}(\mathcal{M})\}$$

$$\mathrm{cof}(\mathcal{M}) = \max\{\mathfrak{d}, \mathrm{non}(\mathcal{M})\}$$

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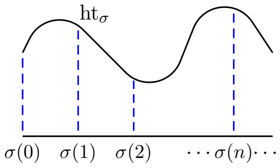

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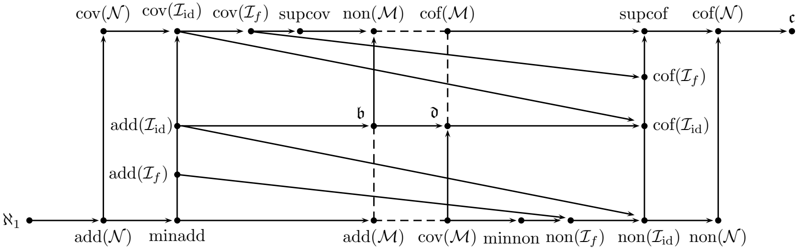

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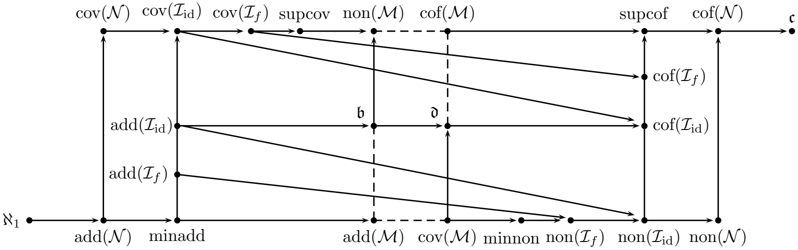
Examples


cov(N)
cov(I_id)
cov(I_f)
supcov
non(M)
cof(M)
supcof
cof(N)
c
cof(I_f)
b
d
add(I_id)
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ℵ_1
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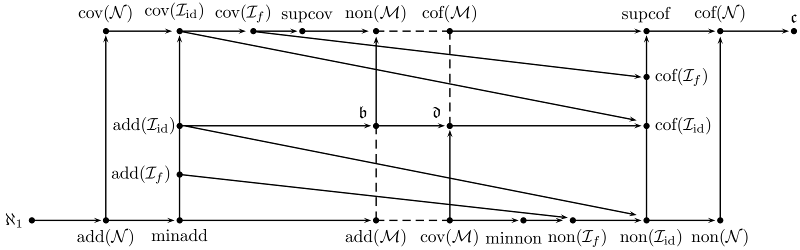

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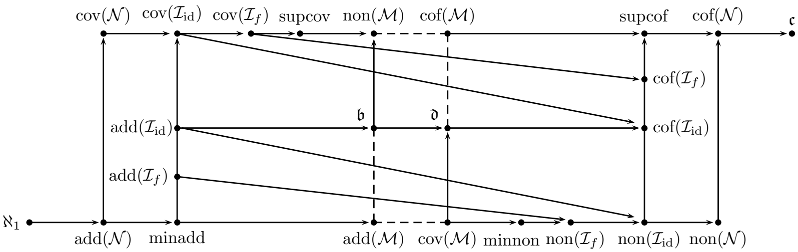

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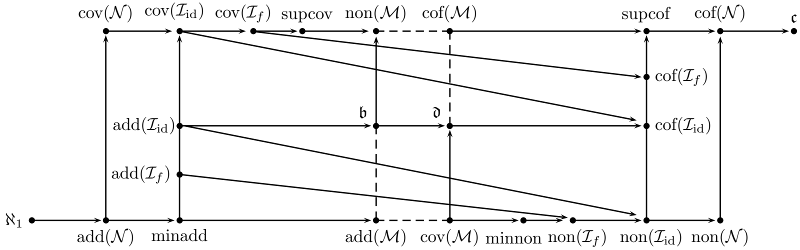

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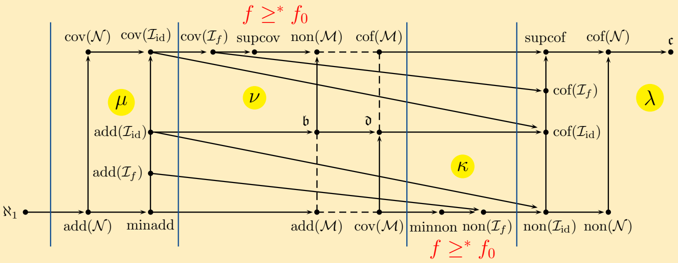
Theorem (C. and Mejía)

Let $\mu \leq \nu \leq \kappa$ be uncountable regular cardinals and $\lambda \geq \kappa$ a cardinal such that $\lambda^{<\mu} = \lambda$. Then there is a function $f_0 \in \omega^\omega$ and a ccc poset forcing



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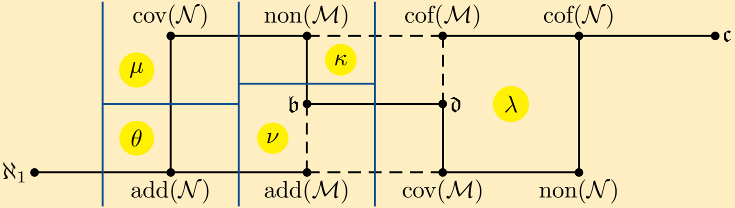
Theorem(Goldstern and Mejía and Shelah 2016)

Let $\theta \leq \mu \leq \nu = \nu^{\aleph_0} \leq \kappa = \kappa^{\aleph_0}$ be uncountable regular cardinals and $\kappa < \lambda$ a cardinal such that $\lambda^{<\kappa} \leq 2^\kappa$ and assume $\mathfrak{b} = \mathfrak{d} = \nu$. Then there is a ccc poset forcing



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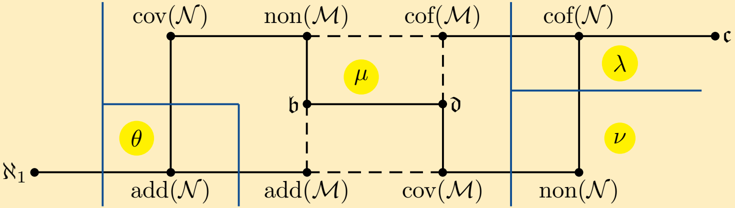

Questions

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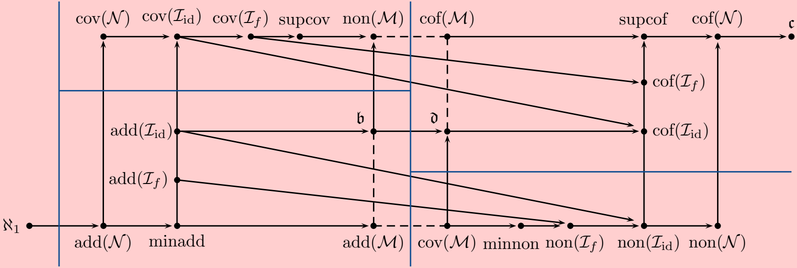
Is it consistent that $\mathrm{add}(\mathcal{I}_f) < \mathrm{cov}(\mathcal{I}_f) < \mathrm{non}(\mathcal{I}_f) < \mathrm{cof}(\mathcal{I}_f)$ for all increasing function f?



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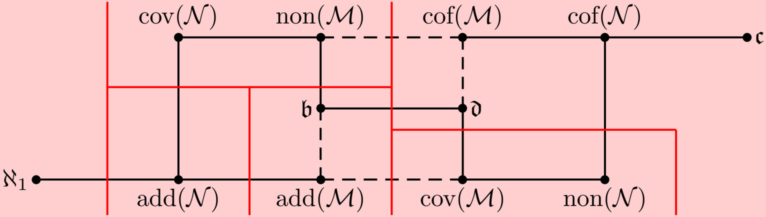

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Theorem(Mejía 2019)

If $\mathbb{P}$ has $\mathfrak{p}$-cc then any subset of $\mathbb{P}$ is *uf*-linked iff it is *Fr*-linked.

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Theorem (Brendle and C. and Mejía)

If θ is an uncountable regular cardinal then any θ-Fr-Knaster poset preserves all the θ-strongly unbounded families from the ground model.

$<\theta$-ultrafilter-extendable matrix iteration

For γ, π ordinals, in the ground model V we construct a matrix iteration $\mathbb{P} = \langle \mathbb{P}_{\alpha,\xi}, \dot{\mathbb{Q}}_{\alpha,\xi} : \alpha \leq \gamma, -1 \leq \xi \leq \pi \rangle$ such that

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(iv) $\mathbb{P}_{\alpha,\xi+1} = \mathbb{P}_{\alpha,\xi} * \dot{\mathbb{Q}}_{\alpha,\xi}$,

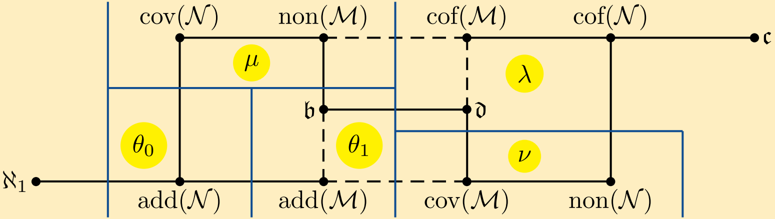
$$\dot{\mathbb{Q}}_{\alpha,\xi} := \begin{cases} \dot{\mathbb{Q}}_\xi & \text{if } \alpha \geq \Delta(\xi), \\ \mathbb{1} & \text{otherwise.} \end{cases}$$

(v) for ξ limit, $\mathbb{P}_{\alpha,\xi} = \mathrm{limdir}_{\eta,\xi} \mathbb{P}_{\alpha,\eta}$.

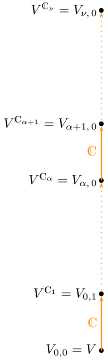
Applications

Theorem(Brendle and C. and Mejía)

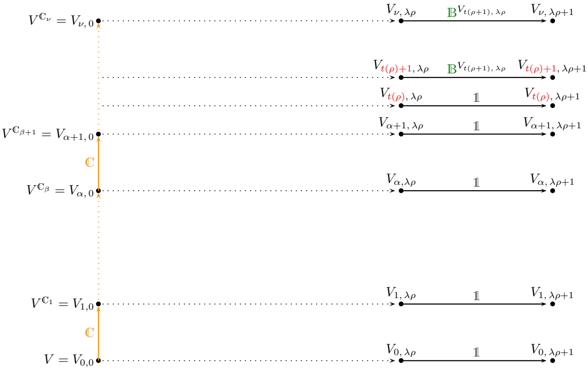
Let $\theta_0 \leq \theta_1 \leq \mu \leq \nu$ be uncountable regular cardinals and let λ be a cardinal such that $\nu \leq \lambda = \lambda^{<\theta_1}$. Then, there is a ccc poset forcing



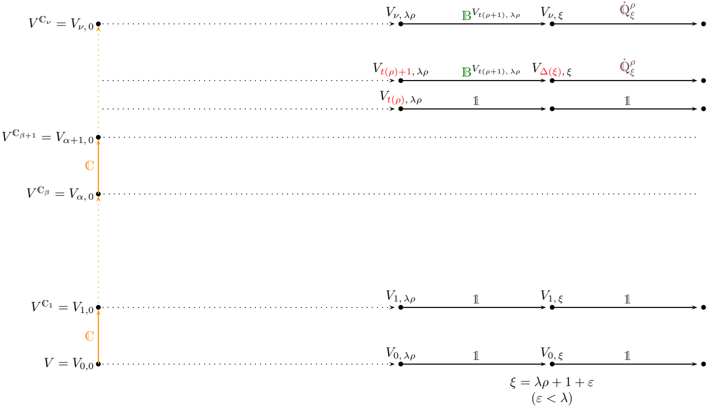
Sketch of the proof



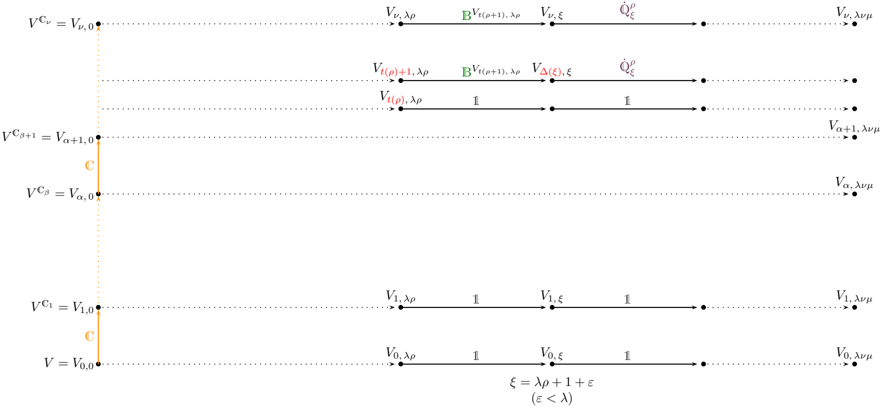
Sketch of the proof



Sketch of the proof



Sketch of the proof



Matrix iteration

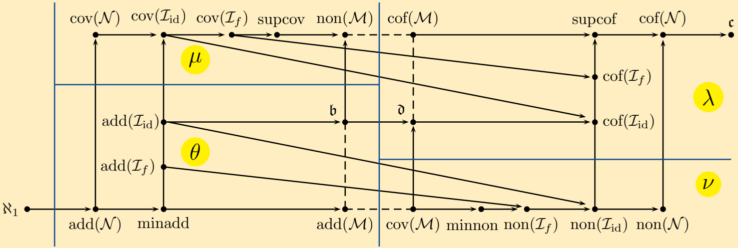
Theorem(Brendle and C. and Mejía)

Let θ be an uncountable regular cardinal and $\mathbb{P}_{\gamma,\pi}$ a $< \theta$-uf-extendable matrix iteration. Then $\mathbb{P}_{\gamma,\pi}$ is θ-uf-Knaster. In particular, it preserves any θ-strongly unbounded family.

Applications

Corollary(Brendle and C. and Mejía)

Let $\theta \leq \mu \leq \nu$ be uncountable regular cardinals and let $\lambda \geq \nu$ be a cardinal such that $\lambda^{<\theta} = \lambda$. Then, there is a ccc poset forcing



Open Problems

Is it consistent with ZFC that

(a) $\mathrm{add}(\mathcal{I}_f) < \mathrm{non}(\mathcal{I}_f) < \mathrm{cov}(\mathcal{I}_f) < \mathrm{cof}(\mathcal{I}_f)$ for any increasing $f : \omega \to \omega$?

Open Problems

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(b) $\mathrm{add}(\mathcal{N}) < \mathrm{non}(\mathcal{N}) < \mathrm{cov}(\mathcal{N}) < \mathrm{cof}(\mathcal{N})$?

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(b) $\mathrm{add}(\mathcal{N}) < \mathrm{non}(\mathcal{N}) < \mathrm{cov}(\mathcal{N}) < \mathrm{cof}(\mathcal{N})$?

(c) $\mathrm{add}(\mathcal{M}) < \mathrm{non}(\mathcal{M}) < \mathrm{cov}(\mathcal{M}) < \mathrm{cof}(\mathcal{M})$?

Open Problems

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(c) $\mathrm{add}(\mathcal{M}) < \mathrm{non}(\mathcal{M}) < \mathrm{cov}(\mathcal{M}) < \mathrm{cof}(\mathcal{M})$?

(d) $\mathrm{add}(\mathcal{SN}) < \mathrm{cov}(\mathcal{SN}) < \mathrm{non}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN})$?

Open Problems

Is it consistent with ZFC that

(a) $\mathrm{add}(\mathcal{I}_f) < \mathrm{non}(\mathcal{I}_f) < \mathrm{cov}(\mathcal{I}_f) < \mathrm{cof}(\mathcal{I}_f)$ for any increasing $f : \omega \to \omega$?

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(d) $\mathrm{add}(\mathcal{SN}) < \mathrm{cov}(\mathcal{SN}) < \mathrm{non}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN})$?

(f) $\mathrm{add}(\mathcal{SN}) < \mathrm{non}(\mathcal{SN}) < \mathrm{cov}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN})$?

Thank you for your attention!

ご清聴
ありがとう
ございました